The role of electron-screening deformations in solar nuclear fusion reactions and the solar neutrino puzzle

Theodore E. Liolios *
Department of Theoretical Physics, University of Thessaloniki, Thessaloniki 54006, Greece
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Abstract

Thermonuclear fusion reaction rates in the stellar plasma are enhanced by the presence of the electron cloud that screens fusing nuclei. The present work studies the influence of electron screening deformations on stellar reaction rates in the framework of the Debye-Hückel model. These electron-ion cloud deformations, assumed here to be static and axially symmetric, are shown to influence the solar neutrino fluxes of the pp and the CNO chains. Moreover, a strong magnetic field is suggested as a plausible source of screening deformations in the sun and the corresponding (magnetized) screening factor is analytically calculated. In such a non-standard solar model a field of $B = 4.7 \times 10^{10} G$ accelerates the pp reaction by roughly 1.7% while the standard solar model neutrino fluxes undergo a correction of $\sim 5\%$ (N, O, B^8) and $\sim 2\%$ (Be^7), respectively. However, the confinement of such a strong magnetic field to the solar energy production region remains as an inherent weakness of this model.

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*theoliol@physics.auth.gr

I. INTRODUCTION

As the quest for the solution to the solar neutrino puzzle continues, all the parameters associated with the production and detection of solar neutrinos are exhaustively investigated. Naturally, stellar thermonuclear reactions have attracted a lot of attention in the nuclear physics community since not only do they govern the evolutionary stages of a star but also they can possibly hide the solution to the notorious discrepancy between theoretically and experimentally calculated solar neutrino fluxes.

The effects of the electron-ion screening on stellar fusion cross sections has been extensively studied [1] [2] [3] [4], especially in relation to the solar neutrino puzzle. Various screening prescriptions have been suggested, each of which has some inherent inadequacies. Salpeter's [1] weak screening formula can be safely used in the study of *pp* solar fusion reactions where the weak screening regime is obeyed, though a recent study showed [5] that, for most practical purposes, it can reasonably be applied to other solar reactions as well. On the other hand, the formula given by Mitler [3] is considered the most reliable as it describes fairly well all the screening regimes, weak screening (WES) , intermediate (IS) and strong screening (SS), though the assumption of a constant electron density around the fusing nuclei is rather arbitrary.

As regards the solar neutrino puzzle, the prevalent belief is that, under the present standard solar model (SSM), the screening effect is under control. However, various non-standard solar models have been proposed, in an attempt to explain the discrepancy between the theoretical and experimental neutrino fluxes. Some of them are plausible, while others are exaggerated. Among those attempts, the existence of a primordial magnetic field in the solar interior has received relatively little attention, especially in connection with the screening effect. This lack of interest stems from the disheartening fact that a large magnetic field (of the order of $\sim 10^9$ Gauss) was shown [6] to increase the predicted event rate of the Cl^{37} experiment by a factor of two. As the magnetic field has the opposite effect from the one that is desired, no further investigation has been made by the same author [7]. Admittedly, some other studies appeared [8] which showed that a combination of differential rotation and strong magnetic fields in the sun can actually reduce the Cl^{37} signal but none of them has been adapted as a component of the SSM. Nevertheless, as it will turn out in the study that follows, a strong magnetic field can influence the solar neutrino fluxes by accelerating the fusion reactions that generate them.

It is well known that strong magnetic fields ($B >> 10^9 G$) compress hydrogen atoms both perpendicular and parallel to the field direction. The magnetic field ties the electrons to the field lines so that their response to a Coulomb attraction is essentially restricted to a one-dimensional motion parallel to the field. It is therefore plausible to assume that the presence of such a field would modify the screening effect of the electron cloud just as it happens on the surface of neutron stars. Such screening deformations can be studied qualitatively by means of the Debye-Hückel model. Actually, in the presence of a large magnetic field the Debye–Hückel radius which represents a spherical distribution of the electron cloud around the nuclei will become orientation dependent. This effects causes, inevitably, reaction rates and neutrino fluxes to be orientation dependent themselves. Moreover, a quantitative study is also possible, by using a recently obtained [13] screening potential which has been used in the study of pp fusion reactions in a superstrong magnetic field.

The purpose of this paper is to investigate the dependence of thermonuclear fusion reaction rates on the deformations of the ionic-electron cloud that screens the reacting nuclei and its possible consequences on solar neutrino fluxes. The layout of the paper is as follows: In §II the Debye-Hückel potential is reviewed and the advantages and disadvantages of its use in the study of solar nuclear reactions are discussed. In §III the formalism for an axially deformed screening cloud is established, underlining its effects on the pp reaction rates and the associated pp neutrino fluxes. A quantitative study of strongly magnetized pp solar reaction is given in §IV while in §V there is given a measure of the uncertainty of the solar neutrino fluxes due to the presence of such a strong field. Finally §VI summarizes the main results of the present work.

II. ADVANTAGES AND DISADVANTAGES OF THE DEBYE-HÜCKEL POTENTIAL

In the stellar plasma gravitational compression and quantum mechanical tunneling combine in order to achieve the classically impossible fusion between light nuclei. The electron gas that surrounds the nuclei acts as a catalyst in the reaction, by lowering the repulsive Coulomb barrier which prevents atomic nuclei from approaching each other.

In the framework of the Debye-Hückel model each nuclei is assumed to polarize its neighborhood creating a spherically symmetric but inhomogeneously charged ionic cloud around it. In this model, which is described in all plasma textbooks, the potential $V(\mathbf{r})$ of a given nucleus of charge Z_1 is related to the charge density $\rho(r)$ by the Poisson equation:

$$\nabla^2 V(\mathbf{r}) = -4\pi\rho(\mathbf{r}) = -4\pi\left[Z_1 e\delta(\mathbf{r}) + \sum_i Z_i n_i e - n_e e\right] \tag{1}$$

where the summation is carried over all particles except electrons. The quantity $\delta(\mathbf{r})$ accounts for nuclear charge deformations which are usually considered negligible.

By assuming statistical equilibrium the $i-$ion number density n_i and the electron number density n_e are related to their mean values by the Boltzmann's formula:

$$n_i(\mathbf{r}) = \overline{n_i}\exp\left[-\frac{Z_i eV(\mathbf{r})}{kT}\right], \qquad n_e(\mathbf{r}) = \overline{n_e}\exp\left[\frac{eV(\mathbf{r})}{kT}\right] \tag{2}$$

where k, T, e, are the Boltzmann's constant, the temperature and the electron charge respectively. A measure of the stellar plasma response to Coulomb interactions is the plasma coupling parameter, which is defined as:

$$\Gamma_{ij} = \frac{Z_i Z_j e^2}{akT} \tag{3}$$

where a is the mean interionic distance. Although the domains of the weak screening (WES) , the intermediate screening (IMS) and strong screening (SS) are not precisely defined , in an ion fluid one can define them as follows:

$$WES : \Gamma_{ij} << 1, \qquad IMS : \Gamma_{ij} \sim 1, \qquad SS : \Gamma_{ij} >> 1 \tag{4}$$

In the WES regime where the Coulomb energy is small compared to the thermal kinetic energy, the number densities can be expanded so that:

$$n_i(\mathbf{r}) = \overline{n_i}\left[1 - \frac{Z_i eV(\mathbf{r})}{kT}\right], \qquad n_e(\mathbf{r}) = \overline{n_e}\left[1 + \frac{eV(\mathbf{r})}{kT}\right] \tag{5}$$

By substituting (5) into Eq.(1) we obtain for large r:

$$\nabla^2 V(r) = r_D^{-2} V(r) \tag{6}$$

with

$$r_D^{-2} = \frac{4\pi e^2}{kT}\left(\sum_i Z_i^2 n_i + n_e \theta_e\right) \tag{7}$$

where r_D is the Debye-Hückel radius and θ_e is the electron degeneracy factor. The solution of (6) is a Yukawa potential which acts in the vicinity of the ion $Z_1 e$ and has the form:

$$V_D(r) = \frac{Z_1 e}{r} \exp\left(-\frac{r}{r_D}\right) \tag{8}$$

It is now obvious that the assumption $\Gamma_{ij} << 1$ of the WES regime for the proton-atom reaction channel between the test nuclei $Z_2 e$ and the generators of the above potential, i.e. nuclei $Z_1 e$ plus ionic cloud, can be written:

$$\frac{Z_1 Z_2 e^2}{r_D kT} << 1 \tag{9}$$

The above condition has generated a lot of controversy in the study of solar fusion cross sections as it is violated for solar nuclear reactions with $Z_1 Z_2$ larger than unity.

Although the Debye-Hückel potential has been used extensively in the study of stellar plasma it has some inherent inadequacies.. An interesting fact is that according to the above model for small r the ion density now vanishes while the electron density diverges. By noting this, Mitler [3] assumed the above model to be valid only beyond some radius r_1 while for shorter distances he assumed the that the electron density around the nucleus is constant and equal to the mean electron density in the plasma, $n_e(\infty)$.

However this is also an oversimplification, especially for the heavier nuclei of astrophysical interest where it underestimates the electron charge density around the ion. For instance near a Be^7 nucleus in central solar conditions we have a density , roughly $3.8 e n_e(\infty)$.

Moreover, the model in question is forced to undergo another compromise when calculating the thermalized cross section $\langle \sigma v \rangle^{sc}$which appears in the screened reaction rate between nuclei i and j :

$$r_{ij}^{sc} = (1 + \delta_{ij})^{-1} n_i n_j \langle \sigma v \rangle^{sc} \tag{10}$$

where n_i, n_j are the number densities of nuclei i and j respectively, and δ_{ij} is the Kronecker delta. As the WKB integral involved cannot be found analytically, we inevitably resort to a linear approximation of potential (8) , that is :

$$V_D\left(r\right) \simeq \frac{Z_1 e}{r}\left(1 - \frac{r}{r_D}\right) \tag{11}$$

If we don't resort to that approximation and take into account non-linearities [9] of the Debye-Hückel potential the thermalized cross section per pair of particles is

$$\langle \sigma v \rangle^{sc} = \sqrt{\frac{8}{\mu\pi}}\left(kT\right)^{-\frac{3}{2}} f_0\left(E_0^{sc}\right) \int_0^\infty S\left(E\right) \exp\left[-\frac{E}{kT} - 2\pi n\left(E\right)\right] dE \tag{12}$$

where the screening correction factor f_0 which is now evaluated at the most probable energy of the screened interaction, is given by:

$$\ln f_0\left(E_0^{sc}\right) = x\pi n\left(E_0^{sc}\right) \tag{13}$$

The quantity x is the ratio of the classical turning point and the D-H radius

$$x = \frac{r_c}{r_D} \tag{14}$$

and is given by the solution of the equation:

$$x e^x \xi^{2/3}\left(x\right) = \frac{1180\left(Z_1 Z_2\right)^{1/3}}{\left(A T_6^2\right)^{1/3} \; r_{D(fm)}} \tag{15}$$

where $\xi\left(x\right)$:

$$\xi\left(x\right) = e^{-x}\left(1 + \frac{x}{2} + \frac{x^2}{16} + \frac{x^3}{48} - \frac{3x^4}{1024} + \frac{13x^5}{5120} - \frac{73x^6}{49152} + ...\right) \tag{16}$$

Note that the most probable energy of interaction is now:

$$E_0^{sc} = 1.220 \cdot \left(Z_1^2 Z_2^2 A T_6^2\right)^{1/3} \xi^{2/3}\left(x\right) \;\; keV \tag{17}$$

where A is the reduced mass of the scattering nuclei in amu.

The above formulae take into account non-linear screening effects of the DH potential and were shown to add a negligible contribution to Salpeter's WES prescription.. However, whenever the WES condition is challenged one should always investigate non-linear corrections as will be shown in the study that follows.

III. QUALITATIVE STUDY OF SCREENING DEFORMATIONS

In the adiabatic model the target and the projectile nuclei are assumed to be surrounded by a static, spherical electron cloud, whose electron charge density falls off exponentially with respect to the distance from the center of the cloud which is the nucleus itself. In central solar conditions the mean ion velocity $\langle u_i \rangle = (8kT/\pi\mu)^{1/2}$ is roughly [4] two hundred times smaller than the mean electron velocity $p_e^2/2m_e \simeq (3/2)\, kT$ thus justifying the fact that as the nucleus moves the electron cloud has enough time to re-arrange itself so that it practically screens the nucleus at all times. However oscillations of the ionic cloud are inevitable due to their speed which is much lower than that of the electrons. For the main sequence stars Mitler [3] showed that the distortion of the common charge clouds has only a small effect on the screening calculations ($\sim 2\%$).

However, the possible presence of a strong magnetic field is bound to cause substantial deformations of the electron cloud by compressing it both parallel and perpendicular to the field direction. Therefore treating the screening cloud as a rigid (albeit inhomogeneous) sphere is an assumption which must by further investigated, especially when primordial magnetic fields are considered.

Studies of heavy nuclei fusion reactions have shown that theoretical predictions of cross section can be greatly improved [10] by assuming rotations and deformations of the fusing nuclei. It is therefore plausible to consider similar effects in the study of screened thermonuclear reactions where the electron cloud is assumed to be deformed. In fact this deformation can be parametrized in the framework of the liquid-drop model so that the Debye-Hückel radius is considered a measure of the electron cloud. The deformed DH radius is now:

$$r_D\left(\theta,\phi\right) = r_D^{(0)}\left[1+\sum_m \beta_m Y_2^m\left(\theta,\phi\right)\right] \tag{18}$$

where $r_D^{(0)}$, takes care of volume conservation and Y_2^m is the usual spherical harmonic function. For simplicity and reasons that will soon become clear only quadrupole deformations will be considered. Moreover, we assume that the deformation is axially symmetric and take the z axis along the axis of symmetry. Disregarding the rotational degree of freedom we obtain the surface shape

$$r_D\left(\theta\right) = r_D^{(0)}\left[1+\beta Y_2^0\left(\cos\theta\right)\right] \tag{19}$$

where the angle θ is measured from the axis of symmetry i.e. the z axis.

Volume conservation is expressed by the relation :

$$\frac{4}{3}\pi r_D^3 = \int_0^{2\pi} d\phi \int_{-1}^{1} d\left(\cos\theta\right) \int_0^{r_D(\theta)} r^2 dr \tag{20}$$

where r_D is given by Eq.(7).

Inserting (19) into the above equation we obtain:

$$r_D^{(0)} = r_D\left\{\frac{1}{2}\int_{-1}^{1}\left[1+\beta\sqrt{\frac{5}{16\pi}}\left(3u^2-1\right)\right]^3 du\right\}^{-1/3} \tag{21}$$

Note that $r_D^{(0)}(\beta) \simeq r_D^{(0)}(-\beta)$.

Therefore for acceptable values of the free parameter β the orientation dependent radius is related to the DH radius r_D of the undeformed electron cloud by means of:

$$r_D(\theta;\beta) = r_D \left\{ \frac{1}{2} \int_{-1}^{1} \left[1 + \beta \sqrt{\frac{5}{16\pi}} \left(3u^2 - 1 \right) \right]^3 du \right\}^{-1/3} \left[1 + \beta \sqrt{\frac{5}{16\pi}} \left(3\cos^2\theta - 1 \right) \right] \quad (22)$$

The two principal semi-axes of the spheroid cloud are now:

A single axis, parallel to the z-axis:

$$r_D(\theta = 0;\beta) = r_D \left\{ \frac{1}{2} \int_{-1}^{1} \left[1 + \beta \sqrt{\frac{5}{16\pi}} \left(3u^2 - 1 \right) \right]^3 du \right\}^{-1/3} \left(1 + \beta \sqrt{\frac{5}{4\pi}} \right) \quad (23)$$

A double axis, perpendicular to the z-axis:

$$r_D\left(\theta = \frac{\pi}{2};\beta\right) = r_D \left\{ \frac{1}{2} \int_{-1}^{1} \left[1 + \beta \sqrt{\frac{5}{16\pi}} \left(3u^2 - 1 \right) \right]^3 du \right\}^{-1/3} \left(1 - \beta \sqrt{\frac{5}{16\pi}} \right) \quad (24)$$

The parameter β is actually a measure of the cloud deformation as it is related to the difference in length between the single and the double axis measured in units of $r_D^{(0)}$:

$$\beta = \frac{2}{3} \sqrt{\frac{4\pi}{5}} \frac{\Delta r_D}{r_D^{(0)}} \quad (25)$$

For $\beta > 0$ the single axis is larger than the double axis and the cloud is a prolate spheroid, that is cigar-shaped.. For $\beta < 0$ the single axis is smaller than the double axis and the cloud is an oblate spheroid i.e. disk-shaped..

Note that the weak screening approximation restricts the possible values of β. A reasonable assumption which stems from Eq.(9) is that in the WES regime the following relation must be fulfilled:

$$\frac{Z_1 Z_2 e^2}{r_D(\theta)\, kT} \leq 0.1 \quad (26)$$

In solar conditions for the pp reaction where the use of the WES formalism is incontrovertible we have

$$\frac{e^2}{r_D kT} \simeq 0.05 \quad (27)$$

Therefore , for all orientations, the following inequality must hold :

$$-0.8 \leq \beta \leq 0.8 \quad (28)$$

where we have disregarded the contribution of volume conservation which is always less than 5%. The deformation parameter can now take all the above values without violating the WES condition (9) , thus rendering the use of the deformed screening formalism legitimate.

If we take into account the nuclear potential $V_N(r)$ then the total potential of the reaction is

$$V(r;\theta;\beta) = V_N(r) + V_D(r;\theta;\beta) + \frac{\hbar^2}{2\mu r^2} l(l+1) \tag{29}$$

where the centrifugal term is assumed to be independent of the orientation. This assumption is immaterial here as we will only consider very low-energy reactions where s-interactions dominate. In that case the orientation dependent cross section of the nuclear fusion reaction is given by:

$$\sigma(E;\theta;\beta) = \frac{S(E)}{E} P(E;\theta;\beta) \tag{30}$$

where

$$P(E;\theta;\beta) = \exp\left[-\frac{2\sqrt{2\mu}}{\hbar}\int_R^{r_c(\theta;\beta)} \sqrt{V_D(r;\theta;\beta) - E dr}\right] \tag{31}$$

and $r_c(\theta;\beta)$ is the classical turning point given by

$$V_D(r_c;\theta;\beta) = E \tag{32}$$

We can now apply the non-linear screening formalism described in the introduction, bearing in mind that the quantity x given by Eq.(14) is now orientation dependent, $x(\theta;\beta)$, and so is the classical turning point

$$x(\theta;\beta) = \frac{r_c(\theta;\beta)}{r_D(\theta;\beta)} \tag{33}$$

Moreover, since the parameter $\xi(\theta)$ depends on the angle at which the projectile enters the electron cloud the Gamow peak is shifted too so that:

$$E_0^{sc}(\theta;\beta) = 1.220 \cdot \left(Z_1^2 Z_2^2 A T_6^2\right)^{1/3} \xi^{2/3}(x(\theta;\beta)) \ \ keV \tag{34}$$

Finally the reaction rates of Eq.(10) are modified through the screening factor which now reads

$$\ln f_0(\theta;\beta) = x(\theta;\beta)\,\pi n\left(E_0^{sc}(\theta;\beta)\right) \tag{35}$$

Note that the influence of the angle θ on the thermonuclear reaction is introduced through the solution of Eq.(15) which yields an angle dependent $x(\theta;\beta)$. The rest of the non-linear screening formalism remains intact.

The thermonuclear reaction which can be studied safely by means of the above formalism is the one that dominates the solar neutrino production namely: $H^1(p,e^+\nu_e)H^2$. For that reaction, in the undeformed weak screening case, it turns out that in the region of the maximum energy production $R = 0.09R_{\odot}$ the Gamow peak is $E_0 = 5.599\,keV$, the classical turning point is roughly $r_c = 0.01r_D$, while the WES enhancement factor is to good approximation: $f_0^{wes} = 1.049$.

Actually, non-linear screening correction can now relax. Along the whole profile of orientations and β parameters the contribution of higher order terms to the shifts of the screening corrections and the Gamow peak has been found negligible. Hence, screening deformation effects can be simply represented by Salpeter's [1] WES formula

$$f_B(\theta;\beta) = \exp\left(\frac{e^2}{kTr_D(\theta)}\right) \tag{36}$$

where the DH radius $r_D(\theta)$ is now orientation dependent. Finally we obtain:

$$f_B(\theta;\beta) = \left(f_0^{wes}\right)^{g^{-1}(\theta;\beta,)} \tag{37}$$

where $g(\theta;\beta)$ is the ratio $r_D(\theta;\beta)/r_D$:

$$g(\theta;\beta) = \left\{\frac{1}{2}\int_{-1}^{1}\left[1+\beta\sqrt{\frac{5}{16\pi}}\left(3u^2-1\right)\right]^3 du\right\}^{-1/3}\left[1+\beta\sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta-1\right)\right] \tag{38}$$

In Fig.1 the orientation dependent DH radius $r_D(\theta;\beta)$, measured in units of r_D, is plotted in cartesian coordinates with respect to the azimuthal angle θ and the deformation parameter β . The effect is calculated for the *pp* reaction in the region of the maximum energy production $R = 0.09R_{\odot}$ where $r_D = 25719fm$. It is obvious that along the axis of symmetry of the cloud, z-axis $(\theta = 0)$, a positive β parameter "stretches out" the ionic cloud (a prolate spheroid shape) while a negative β parameter "sucks in" the cloud (an oblate spheroid shape). As one would expect, for both positive and negative parameters the larger the absolute value of β the more pronounced the deformation.

An equally enlightening picture of the catalytic influence of screening deformations on the *pp* reactions is obtained through Fig.2 which describes the same effect in polar coordinates. Note that both Fig.1 and Fig.2 actually represent the deformation factor $g(\theta;\beta)$ while the corresponding classical turning point is $r_c = 261g(\theta;\beta)\,fm$. For example for a $\beta = 0.8$ deformation a proton cruising along the z-axis in the plasma with an energy equal to the Gamow peak $E_0 = 5.56\ keV$ will come up against the potential wall at a distance roughly 1.43 times further than in the undeformed case ($r_c = 261fm$), that is $r_c(\theta = 0, \beta = 0.8) = 373fm$. On the other hand for a $\beta = -0.8$ the classical turning point is reduced to $r_c(\theta = 0, \beta = -0.8) = 123fm$. Note that both Fig.1 and Fig.2 actually represent the deformation factor $g(\theta;\beta)$ while the classical turning point is $r_c = 261g(\theta;\beta)\,fm$.

The screened Coulomb potential $V(r;\theta;\beta)$ can be visualized by means of Fig.3 where we have plotted in polar coordinates the deformed shapes of the potential at a distance $r = r_D$ from the origin. At that distance the potential contours $V_D(r_D;\theta;\beta)$ of Fig.3 are only a function of the orientation. Hence, as a proton enters the ionic cloud of the Hydrogen atom on its way to fusion, according to the angle at which it enters the cloud it will experience a different (thicker or thinner) potential wall . The thicker the wall, the most improbable the reaction and of course the smaller the reaction rate.

The orientation dependent acceleration of the reaction is displayed in Fig.4. where the screening factor is plotted with respect to the azimuthal angle and the deformation parameter for the pp reaction at $R = 0.09 R_{\odot}$. The reaction rate can be 1.1 times faster if the proton enters a disk-shaped cloud ($\beta = -0.8$) at zero angle. On the other hand a much slower reaction is obtained for a cigar-shaped ionic cloud (almost no enhancement at all for $\beta = 0.8$).

The impact of screening deformation on the neutrino fluxes will be discussed in a more quantitative way in the section that follows.

IV. QUANTITATIVE STUDY OF MAGNETICALLY INDUCED SCREENING DEFORMATIONS

The investigation in the previous section would be purely academic if there was not a mechanism that can induce screening deformations such as a strong magnetic field.

In the previous section the effects of screening deformations were studied in the framework of the liquid-drop model without analyzing the source of the deformation. Naturally, the most plausible source of deformations is a strong magnetic field. Two very successful methods for studying magnetized proton-proton fusion reactions is the Hartree-Fock approximation [11] [12] and the adiabatic approximation [13] [14], the latter being briefly reviewed here.

In the presence of a magnetic field the Hamiltonian of the electron in a hydorogen-like atom is :

$$H = \frac{\mathbf{p}^2}{2m} - \frac{Ze^2}{r} - \mu \mathbf{B} \tag{39}$$

where m is the mass of the nucleus assumed to be infinite, $\mathbf{B}$ is the magnetic field and μ is the magnetic moment. In the above equation the vector potential $\mathbf{A}$ is included in the momentum operator:

$$\mathbf{p} = -i\hbar\nabla - (e/c)\,\mathbf{A} \tag{40}$$

To simplify the problem we usually choose a magnetic field along the z axis of the frame of reference whose origin coincides with the center of the nucleus. Moreover, we choose such a gauge for the vector potential so that in cylindrical coordinates,

$$\mathbf{A} = \frac{B\rho}{2}\widehat{\phi} \tag{41}$$

Under those assumptions the Schroedinger equation for the ground state of the hydrogen atom is written as:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{i\hbar}{2mc}B\,|e|\,\frac{\partial}{\partial\phi} + \frac{1}{8}\frac{e^2}{mc^2}B^2\rho^2 - \frac{Ze^2}{r}\right]\Psi_{nm0} = E_{nm0}\Psi_{nm0} \tag{42}$$

In the above equation we have assumed that the electron spins are antiparallel to the field and we therefore ignore the spin degree of freedom.

In the absence of a Coulomb field the electron motion perpendicular to a uniform magnetic field is quantized into Landau orbits . The radius of the lower orbit is called cyclotron radius and is defined as:

$$\widehat{\rho} = \sqrt{\frac{\hbar \mathbf{c}}{|\mathbf{e}|\, B}} = b^{-1/2} a_0 \tag{43}$$

where a_0 is the Bohr radius, $b = B/B_0$ and $B_0 = 2.35 \times 10^9 G$. The corresponding cyclotron energy is

$$\hbar\omega_e = \frac{\hbar\, |\mathbf{e}|\, B}{m_e c} = 11.57 B_{12}\, (keV) \tag{44}$$

where B_{12} is the magnetic field measured in units of $10^{12} G$. Moreover, the higher Landau levels which are actually electron distribution maxima are given by:

$$\rho_m = (2m+1)^{1/2}\, \widehat{\rho} \tag{45}$$

If the field is sufficiently strong so that:

$$\frac{Ze^2}{\widehat{\rho}} << \hbar\omega \tag{46}$$

then the adiabatic approximation can be employed where the Coulomb potential can be treated as a perturbation to the unmagnetized problem. In that approximation the ground state solutions $(n = 0, l = 0, m = 0)$ of the Schroedinger equation are given as:

$$\Psi\,(\rho, \phi, z) = R\,(\rho, \phi)\, Z\,(z) \tag{47}$$

where $R\,(\rho, \phi)$ are the usual Landau functions [15] and $Z\,(z)$ a function to be determined.

In a relatively recent work a variational method was employed for the study of proton-proton fusion reactions in the surface of neutron stars where it is widely believed that a superstrong magnetic field of the order of $10^{12} G$ exists [16]. Under such extreme conditions the electron-screening cloud is deformed in the sense that it becomes compressed perpendicular and parallel to the magnetic field. That work [13] using a gravitational analog yielded a screening potential for the strongly magnetized Hydrogen atom:

$$\Phi_\alpha\,(\rho, z) = \frac{e^2}{\widehat{\rho}} \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{\exp\left[-\frac{1}{2}\left(\frac{\overline{\rho}^2}{1+u} + \frac{\overline{z}^2}{\alpha^2+u}\right)\right]}{(1+u)\sqrt{\alpha^2+u}} du \tag{48}$$

The natural length unit in the above formula is of course the cyclotron radius so that $\overline{\rho} = \rho/\widehat{\rho}$, $\overline{z} = z/\widehat{\rho}$, and α is a parameter to be determined by the variational method. The above formula was shown to be reliable for very strong fields whereas it becomes inaccurate below the threshold of the intense magnetic field regime given by:

$$B_{IMF} = 4.7 \times 10^9 G. \tag{49}$$

By means of the above potential , we will attempt to investigate the effects of magnetically induced screening deformations on the solar proton-proton fusion reaction and the solar neutrino problem.

Let us assume that a field of $B = 4.7 \times 10^{10} G$ exists in the solar energy production region. This field would generate a screening potential according to the above model. Therefore according to the formalism analyzed in the qualitative study of deformations the cross section will be given by (30) while the potential energy of the proton-proton interaction is:

$$V(\overline{r}, \theta) = \frac{e^2}{\widehat{\rho}\overline{r}} - \Phi_\alpha(\overline{r}\sin\theta, \overline{r}\cos\theta) \tag{50}$$

where $\overline{r} = r/\widehat{\rho}$.

The classical turning point $\overline{r}_c(E;\theta)$ in units of the cyclotron radius $\widehat{\rho}$ will now be given by the equation:

$$\frac{e^2}{\widehat{\rho}\overline{r}_c} - \Phi_\alpha(\overline{r}_c\sin\theta, \overline{r}_c\cos\theta) - E = 0 \tag{51}$$

where E is the interaction energy of the proton-proton reaction.

The penetration factor is now:

$$P_B(E;\theta) = \exp\left[-\frac{2\sqrt{2\mu\widehat{\rho}}\,|e|}{\hbar}\int_R^{r_c(E;\theta)} \sqrt{\frac{1}{\overline{r}} - \widetilde{\Phi}_\alpha(\overline{r}\sin\theta, \overline{r}\cos\theta) - \widetilde{E}d\overline{r}}\right] \tag{52}$$

where $\widetilde{\Phi}_\alpha$ and $\widetilde{E}$ the screening and interaction energies respectively, measured in units of the cyclotron energy $(e^2/\widehat{\rho})$. If we use the numerical values that correspond to the parameters in the above formula we obtain:

$$P_B(E;\theta) = \exp\left[-19.05 B_{12}^{-1/4}\int_R^{r_c(E;\theta)} \sqrt{\frac{1}{\overline{r}} - \widetilde{\Phi}_\alpha(\overline{r}\sin\theta, \overline{r}\cos\theta) - \widetilde{E}d\overline{r}}\right] \tag{53}$$

which corrects the numerical coefficient of the Eq.(44) in ref. [13]. Note that if the coefficient 19.05 is used then all the numerical calculation of ref. [13] are recovered, so that must only be a misprint. To gain an idea of the enhancement of the *pp* reaction caused by the presence of the above magnetic field we can use the penetration factor which actually represents the probability that the reaction will occur. For the unmagnetized unscreened case at $R = 0.09 R_\odot$ we have $\ln P_{B=0}^{NOS} = -9.43$,while for the magnetic field considered the probability is $\ln P_B = -9.41$. This seemingly unimportant difference will turn out to be capable of perturbing the neutrino fluxes.

The magnetically catalyzed (thermalized) cross section $\langle \sigma v \rangle^m$ is now proportional to the integral:

$$\int_0^\infty S(E)\exp\left[-\frac{E}{kT} - \frac{2\sqrt{2\mu\widehat{\rho}}\,|e|}{\hbar}\int_R^{r_c(E;\theta)} \sqrt{\frac{1}{\overline{r}} - \widetilde{\Phi}_\alpha(\overline{r}\sin\theta, \overline{r}\cos\theta) - \widetilde{E}d\overline{r}}\right] dE \tag{54}$$

In the unmagnetized case the screening factor is evaluated at the most probable energy of interaction E_0, which is given by Eq.(17). A crucial issue is the determination of the energy at which the integral will be evaluated. This energy will of course determine the classical turning point $\overline{r}_c(E;\theta)$ and is certain to be lower than the one given by Eq(17) due to the presence of the screening potential which lowers the barrier. By using various most probable interaction energies , including that of the unmagnetized case, we obtain the following table:

Table I here

In order to obtain the screening factor we have to numerically integrate the integral in Eq.(54) which is bound to introduce inaccuracies. Instead we resort to a simpler method which will yield an analytic formula. Studying the variation of the screened and unscreened potential energies with respect to distance (see Fig.5) some very interesting results appear, which are verified by the tabulated results ($Table\, I$) of the numerical solution of Eq. (51). For most reasonable interaction energies the classical turning point is only slightly displaced and it is always much lower than the distance $r = \widehat{\rho}/2$. As it was pointed out [13] the screening potential energy of Eq.(48) for distances shorter than $\widehat{\rho}/2$ can be approximated by its value at the center of the cloud:

$$\Phi_\alpha(0,0) = \frac{e^2}{\widehat{\rho}}\frac{2}{\sqrt{2\pi}}\frac{\ln\left(\alpha+\sqrt{\alpha^2-1}\right)}{\sqrt{\alpha^2-1}} \tag{55}$$

Consequently, for the weakly magnetized proton-proton solar reaction considered we can assume that the Coulomb potential energy e^2/r is displaced by $\Phi(0,0)$ instead of the usual e^2/r_D quantity of the Debye-Hückel model. It is now easy to derive the screening (accelerating) factor of the *pp* reaction. According to Salpeter's approach [1] (see also ref. [17]) for an energy independent screening term the screened reaction rate between two protons in the solar plasma is proportional to the integral:

$$\int_0^\infty E^{1/2}e^{-\frac{E}{kT}}P\left(E+\Phi_\alpha(0,0)\right)\sigma_{nuc}\left(E+\Phi_\alpha(0,0)\right)dE \tag{56}$$

where the cross section $\sigma(E)$ has been written as the product of the penetration factor $P(E)$ times a nuclear factor $\sigma_{nuc}(E)$. The integral can now be written as:

$$\int_{\Phi_\alpha(0,0)}^\infty \left(E^{'}-\Phi_\alpha(0,0)\right)^{1/2}e^{-\frac{E^{'}-\Phi_\alpha(0,0)}{kT}}P\left(E^{'}\right)\sigma_{nuc}\left(E^{'}\right)dE^{'} \tag{57}$$

It is obvious that $\Phi_\alpha(0,0)$ is small when compared to the most effective energy of interaction so $\left(E^{'}-\Phi_\alpha(0,0)\right)^{1/2}\simeq\left(E^{'}\right)^{1/2}$ and the lower limit can be extended to zero. For example for the magnetic field considered here $\Phi_{\alpha=2}(0,0) = 0.075keV$ while for the unmagnetized case the Debye-Hückel screening term is $e^2/r_D = 0.056\,keV$ and the corresponding most effective interaction energy is $E_0 = 5.6keV$. Hence, the magnetically catalyzed screened reaction rate $r_{ij}^m$is related to the unscreened one r_{ij}^{NOS} by:

$$r_{ij}^{m} = f_B r_{ij}^{NOS} \tag{58}$$

where the magnetic screening factor is :

$$f_B = \exp\left(\frac{\Phi_\alpha\left(0,0\right)}{kT}\right) \tag{59}$$

Some comments on the accuracy of the above formula are necessary here. The potential $\Phi_\alpha\left(0,0\right)$ was derived by using a probability density whose accuracy has been pointed out in ref. [13]. In fact for the field considered that density produced ground state binding energies 5% smaller than the more accurate ones produced by Ruder et al [18]. This error must be taken into account when using the magnetic screening factor f_B. For example if we naively assume that this error propagates through our calculations intact then $\Phi_\alpha\left(0,0\right)$ should carry a similar error and the corrected magnetized screening factor should be:

$$f_B = \exp\left(\frac{\Phi_\alpha\left(0,0\right)\left(1\pm 0.05\right)}{kT}\right) \tag{60}$$

Note that the most effective energy of interaction is insensitive to such a small energy shift as $\Phi_\alpha\left(0.0\right)$ and is still be given by Eq.(17) .

V. MAGNETICALLY CATALYZED SOLAR NEUTRINO FLUXES.

In the solar region of maximum energy production ($R/R_\odot = 0.09$) the thermal kinetic energy is $kT = 1.161keV$ while for the *pp* reaction the unmagnetized screening factor is $f_0 = 1.049$. On the other hand for the magnetic field considered the magnetized screening factor is $f_B = 1.067 \pm 0.003$. This corresponds to an acceleration of the (unmagnetized) weakly screened *pp* reaction by roughly 1.7% which in turn reflects on the magnetized cross-section factor given by $S_B = f_B S$. As the principal source of energy in the Sun is the *pp* reaction this acceleration would influence both the solar structure and the neutrino fluxes by reducing the central temperature and density in order to conserve luminosity. (An account of what happens in the sun if the cross-section factor S_{pp} increases can be found in ref. [19]).

Although the magnetically induced deformation assumed here is spherical we can still relate it to the qualitative study of §III by observing that a similar deformation is caused along the z-axis for $\beta \simeq -0.4$

In most solar evolution codes the *pp* screening factor is evaluated by means of Salpeter's formula which has been proved to be valid and accurate in standard conditions. In the magnetized case the quantity f_B should be used. The most proper way to study this factor is to feed it into a solar evolution code and observe the new fluxes and the corresponding Cl^{37} signal. At the moment such a code is unavailable to the author therefore in order to obtain a picture of the uncertainties introduced due to the presence of such a strong field one should use the proportionality formulae [7] [20] which relate neutrino fluxes to screening factors. In order to isolate the *pp* uncertainty, we will assume that except for the *pp* reaction all the other neutrino-producing reactions remain unaffected by the magnetic field . Besides the adiabatic approximation breaks down for nuclei with $Z > 1$ and the field considered in this work because the (already abused) condition (46) is further violated.

For various solar fusion reactions the ratios of the magnetized neutrino fluxes Φ^m to the ones obtained in the WES regime Φ^{WES}, are as follows:

$H^1\left(p, e^+\nu_e\right) H^2$

$$\left(\frac{\Phi^m_{pp}}{\Phi^{WES}_{pp}}\right)_{pp} = \left(\frac{f_B}{f_0}\right)^{0.14} = 1.002 \tag{61}$$

$H^1\left(pe^-, \nu_e\right) H^2$

$$\left(\frac{\Phi^m_{hep}}{\Phi^{WES}_{hep}}\right)_{pp} = \left(\frac{f_B}{f_0}\right)^{-0.08} = 0.998 \tag{62}$$

$Be^7\left(e^-, \nu_e\right) Li^7$:

$$\left(\frac{\Phi^m_{Be^7}}{\Phi^{WES}_{Be^7}}\right)_{pp} = \left(\frac{f_B}{f_0}\right)^{-0.97} = 0.983 \tag{63}$$

$Be^7\left(p, \gamma\right) B^8\left(e^+, \nu_e\right) B^{8*}$:

$$\frac{\Phi^m_{B}}{\Phi^{WES}_{B}} = \left(\frac{f_B}{f_0}\right)^{-2.6} = 0.956 \tag{64}$$

$N^{13}\left(e^+\nu_e\right) C^{13}$ and $O^{15}\left(e^+, \nu_e\right) N^{15}$:

$$\frac{\Phi^B_{N,O}}{\Phi^{WES}_{N,O}} = \left(\frac{f_B}{f_0}\right)^{-22/8} = 0.954 \tag{65}$$

According to the above results, the presence of a reasonably strong magnetic field $\sim 4.7\times10^{10}G$, can perturb the solar neutrino fluxes calculated in the SSM by at least 5% for the N, O and B^8 neutrinos and by less than 1% for the less sensitive pp, hep,and B^7neutrinos.. Such a magnetic field would also perturb the electron cloud around the heavier nuclei involved in neutrino production, thus modifying their screening factors which we arbitrarily considered constant here. However, a more efficient perturbation method should be used for such reactions, one which would not be hampered by the choice of the trial function used in ref. [13].

It is very tempting to investigate what happens if the magnetic field is stronger. In fact, for most reasonable interaction energies (see table I), formula (59) is also valid for a field of $4.7 \times 10^{11}G$. In that case $\Phi_{\alpha=3.79}\left(0,0\right) = 0.167\ keV$ so that the corresponding screening factor is $f_B = 1.154$. It is easy to show that with such a field we can obtain perturbations (reductions) of the SSM fluxes from the order of 9% $\left(Be^7\right)$ up to the order of 22% $\left(B^8\right)$. Adding the effects of the screening factors of the other reactions, which have been disregarded so far, the uncertainties can be dramatic. It seems therefore that the presence of a superstrong magnetic field in the sun can tune the predicted neutrino fluxes in order to reduce the observed deficit.

Admittedly such a superstrong field cannot be easily justified. After the disheartening result [6] that the presence of a strong magnetic ($\sim 10^9 G$) in the solar interior increases the predicted neutrino fluxes (doubles the Cl^{37} signal by increasing the pressure gradient in the sun) few investigators have looked into the matter. This is largely due to some additional arguments which indicate the non-existence of a field larger than $10^9 G$. Such arguments include the limiting strength set by Chandrasekhar and Fermi [22], stability reasons [23] and magnetic buoyancy [24], though there is a very interesting work [8] which argues that a combination of a differential rotation and magnetic field can reduce the Cl^{37} signal opposing the results of ref. [6]

From the present work it is obvious that if a substantial correction to the solar neutrino fluxes is to be made by means of magnetically catalyzed thermonuclear fusion, the magnetic field required must be stronger than $10^{10} G$. Such a field can have been formed by the interstellar magnetic field which was frozen into the matter out of which the sun was formed, or there may be an unspecified mechanism of continuous generation.

VI. CONCLUSIONS

In this work we investigate the response of the proton-proton reaction to electron-ion screening deformations in the solar plasma. Those deformations are qualitatively studied in the framework of the Debye-Hückel model and the results show that they can induce an orientation-dependent thermalized cross section which causes the solar neutrino fluxes to be orientation-dependent themselves..

A strong magnetic field is suggested as a plausible source of screening deformations and an analytic (magnetic) screening factor is obtained which can be used in order to evaluate both the magnetic acceleration of the *pp* reaction and the corresponding magnetically catalyzed solar neutrino fluxes. In such a non-standard solar model a field of $B = 4.7 \times 10^{10} G$ accelerates the *pp* reaction by roughly 1.7% while the standard solar model neutrino fluxes undergo a correction of $\sim 5\%$ (N, O, B^8) and $\sim 2\%$ (Be^7), respectively. If stronger fields are considered the uncertainties become more dramatic.

Although such a strong field cannot be easily justified, it seems that its role in the solar neutrino puzzle should be further investigated, especially with regard to its effect on solar nuclear reactions.

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FIGURE CAPTIONS

Figure 1.

The variation of the orientation dependent DH radius $r_D(\theta;\beta)$, measured in units of r_D, with respect to the azimuthal angle θ and the deformation parameter β in cartesian coordinates. The effect is calculated for the *pp* reaction in the region of the maximum energy production $R = 0.09R_{\odot}$ where $r_D = 25719fm$

Figure 2.

The variation of the orientation dependent DH radius $r_D(\theta;\beta)$, measured in units of r_D, with respect to the azimuthal angle θ and the deformation parameter β in polar coordinates. The effect is calculated for the *pp* reaction in the region of the maximum energy production $R = 0.09R_{\odot}$ where $r_D = 25719fm$.

Figure 3.

The orientation dependent DH potential $V_D(r;\theta;\beta)$, measured in units of the screening term of Eq.(27), at a distance $r = r_D$ with respect to the azimuthal angle θ and the deformation parameter β in polar coordinates. The effect is calculated for the *pp* reaction in the region of the maximum energy production $R = 0.09R_{\odot}$ where $r_D = 25719fm$ and $\frac{e^2}{r_D kT} \simeq 0.056\ keV$.

Figure 4.

The variation of the orientation dependent screening factor $f_B(\theta;\beta)$ with respect to the azimuthal angle θ and the deformation parameter β in polar coordinates. The effect is calculated for the *pp* reaction in the region of the maximum energy production $R = 0.09R_{\odot}$ where $r_D = 25719fm$.

Figure 5.

The potential energies measured in units of $(e^2/\widehat{\rho})$,with respect to the distance u (in units of $\widehat{\rho}$) between the two colliding protons. The dotted line indicates the screening term $\Phi_\alpha(u,\theta)$, the dashed line the unscreened Coulomb potential energy $(1/u)$ and the solid line their difference which is the actual screened potential energy of the reaction.

TABLES

Table I . The magnetized ($\overline{r}_c$) and unmagnetized (r_c) classical turning points for various interaction energies . For the magnetic field considered ($4.7 \times 10^{10} G$) the cyclotron radius is $\widehat{\rho} = 11627 fm$ and the corresponding Coulomb energy is $e^2/\widehat{\rho}$ =0.123 keV.

$E\,(keV)$	5.5	4.5	3.5	2.5	1.5	0.5
$\widetilde{E}\,(e^2/\widehat{\rho})$	44.7	36.58	28.45	20.32	12.19	4.0
$\overline{r}_c\,(\widehat{\rho})$	0.022	0.026	0.034	0.047	0.078	0.214
$\overline{r}_c\,(fm)$	256	302	395	546	907	2523
$r_c\,(fm)$	261	320	411	576	960	2880